MULTIPARTICLE PRODUCTION AT RHIC AND LHC: A CLASSICAL POINT OF VIEW

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We report results of our ongoing nonperturbative numerical study of a classical effective theory describing low-x partons in the central region of a heavy-ion collision. In particular, we give estimates of the initial transverse energies and multiplicities for a wide range of collision regimes, including those at RHIC and at LHC.

In the central-rapidity region of heavy-ion collisions at RHIC and at LHC a combination of very high center-of-mass energy with a very large number of participating valence quarks is expected to lead to a novel regime of QCD, one characterized by a very high parton density. This regime may not be amenable to analysis by conventional methods, such as multiple scattering or classical cascade descriptions, which ignore the coherence of the gluon field [1].

This coherence emerges naturally from the classical effective field theory approach of McLerran and Venugopalan (MV) [2]. If the parton density in the colliding nuclei is high at small x, classical methods are valid. It has been shown recently that a RG-improved generalization of this effective action reproduces several key results in small-x QCD [3].

The model can be summarized as follows. The high-x and the low-x partons are treated separately. The former corresponds to valence quarks and hard sea partons. These high-x partons are considered recoilless sources of color charge. For a large Lorentz-contracted nucleus, this results in a static Gaussian distribution of their color charge density ρ in the transverse plane:

$$P\left([\rho]\right) \propto \exp\left[-\frac{1}{2\Lambda_s^2}\int \mathrm{d}^2 r_t \rho^2(r_t)\right].$$

The variance Λ_s of the color charge distribution is the only dimensional parameter of the model, apart from the linear size L of the nucleus. For central impact parameters, Λ_s can be estimated in terms of single-nucleon structure

functions [4]. It is assumed, in addition, that the nucleus is infinitely thin in the longitudinal direction. Under this simplifying assumption the resulting gauge fields are boost-invariant.

The small x fields are then described by the classical Yang-Mills equations

$$D_\mu F_{\mu\nu} = J_\nu \tag{1}$$

with the random sources on the two light cones: $J_\nu = \sum_{1,2} \delta_{\nu,\pm}\delta(x_\mp)\rho_{1,2}(r_t)$. The two signs correspond to two possible directions of motion along the beam axis z. As shown by Kovner, McLerran and Weigert (KMW) [5], low-x fields in the central region of the collision obey sourceless Yang-Mills equations (this region is in the forward light cone of both nuclei) with the initial conditions in the $A_\tau = 0$ gauge given by

$$A^i = A^i_1 + A^i_2; \qquad A^\pm = \pm\frac{ig}{2}x^\pm[A^i_1, A^i_2]. \tag{2}$$

Here the pure gauge fields $A^i_{1,2}$ are solutions of (1) for each of the two nuclei in the absence of the other nucleus.

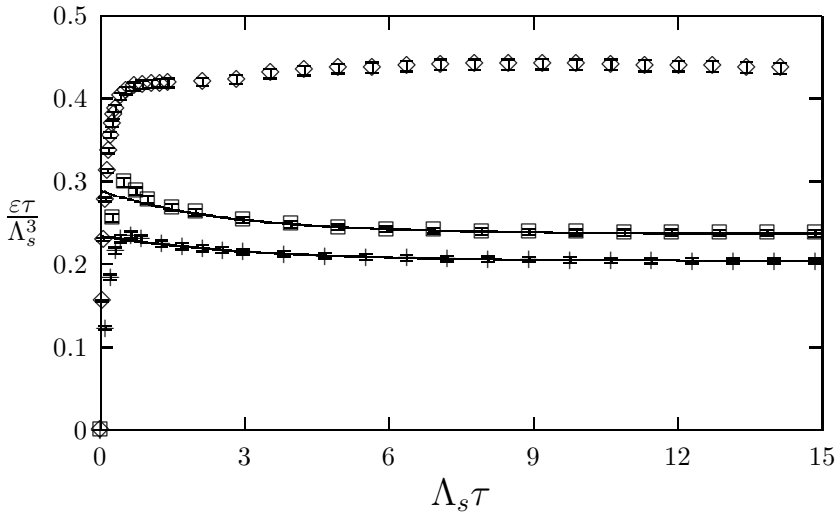

Figure 1: Transverse-plane energy density per unit rapidity versus proper time for the values 5.66 (diamonds), 35.36 (plusses), and 297 (squares) of $\Lambda_s L$.Both the energy density and the proper time are expressed in units of Λ_s. The solid lines are fits of the data to the form $\alpha + \beta\exp(-\gamma\tau)$.

In order to obtain the resulting gluon field configuration at late proper times, one needs to solve (1) with the initial condition (2). Since the latter

depends on the random color source, averages over realizations of the source must be performed. KMW showed that in perturbation theory the gluon number distribution by transverse momentum (per unit rapidity) suffers from an infrared divergence and argued that the distribution must have the form

$$n_{k_\perp} \propto \frac{1}{\alpha_s} \left(\frac{\Lambda_s}{k_\perp}\right)^4 \ln\left(\frac{k_\perp}{\Lambda_s}\right) \tag{3}$$

for $k_\perp \gg \Lambda_s$. The log term clearly indicates that the perturbative description breaks down for $k_\perp \sim \Lambda_s$.

A reliable way to go beyond perturbation theory is to re-formulate the EFT on a lattice by discretizing the transverse plane. The resulting lattice theory can then be solved numerically. We shall not dwell here on the details of the lattice formulation, which is described in detail in Ref. [6]. Keeping in mind that Λ_s and the linear size L of the nucleus are the only physically interesting dimensional parameters of the model[7], we can write any dimensional quantity q as $\Lambda_s^d f_q(\Lambda_s L)$, where d is the dimension of q. All the non-trivial physical information is contained in the dimensionless function $f_q(\Lambda_s L)$. We can estimate the values of the product $\Lambda_s L$ which correspond to key collider experiments. Assuming Au-Au collisions, we take $L = 11.6$ fm (for a square nucleus!) and estimate the standard deviation Λ_s to be 2 GeV for RHIC and 4 GeV for LHC[4]. Also, we have approximately $g = 2$ for energies of interest. The rough estimate is then $\Lambda_s L \approx 120$ for RHIC and $\Lambda_s L \approx 240$ for LHC.

We now proceed to describe the results of our numerical study so far. Our simulations were performed for the SU(2) gauge group, in order to keep the computational costs low.

$\Lambda_s L$	17.68	35.36	70.7	106.06	148.49	212.13	296.98
f_E	.323(4)	.208(4)	.200(5)	.211(1)	.232(1)	.234(2)	.257(5)

Table 1: The function f_E, *i.e.*, the energy per unit transverse area per unit rapidity, expressed in units of Λ_s, tabulated vs $\Lambda_s L$.

We first compute the energy per unit transverse area per unit rapidity, deposited in the central region by the colliding nuclei. As Figure 1 illustrates, this quantity tends to a constant at late proper times. We find that at $\tau \longrightarrow \infty$ the energy density in units of Λ_s depends on the dimensionless parameter $\Lambda_s L$ as described in Table 1. Note the very slow variation of this dimensionless function in the entire range of $\Lambda_s L$ values, which includes both our RHIC and LHC estimates. Using this plot, and assuming, in accordance with Ref. [8], the $(N_c^2 - 1)/N_c$ dependence of the energy on the number of colors N_c, we arrive

at the values of 2700 GeV and of 25000 GeV for the transverse energy per unit rapidity at RHIC and at LHC, respectively [9].

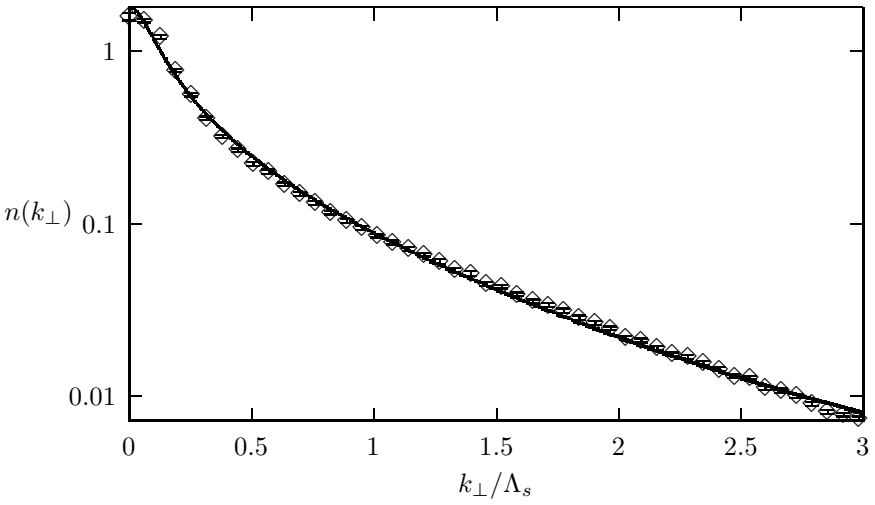

Figure 2: $n(k_\perp) \equiv dN/L^2/d^2k_\perp$ as a function of the gluon momentum $k_\perp$ for $\Lambda_s L = 297$ (diamonds). The solid line is a fit to Bose-Einstein distribution, corresponding to a gas of free particles at the inverse temperature $\beta = (1.28 \pm 0.01)\Lambda_s$ and with the mass $m = (0.092 \pm 0.003)\Lambda_s$.

The number and distribution of produced gluons are of considerable interest as initial data for possible evolution of the gluon gas towards thermal equilibrium [8,10,11]. Strictly speaking, a particle number is only well-defined in a free-field theory, and there is no unique extension of this notion to a general interacting case. For this reason we use two different generalizations of the particle number to an interacting theory, each having the correct free-field limit. We verify that the two definitions agree in the weak-coupling regime corresponding to late proper times in the central region [12].

Our first definition is straightforward. We impose the Coulomb gauge condition in the transverse plane: $\vec{\nabla}_\perp \cdot \vec{A}_\perp = 0$ and and determine the momentum components of the resulting field configuration. Our second definition is based on the behavior of a free-field theory under relaxation. Consider a simple relaxation equation for a field in real space,

$$\partial_t \phi(x) = -\partial V/\partial\phi(x), \tag{4}$$

where t is the relaxation time (not to be confused with real or proper time) and

V is the potential. It is then easy to derive the following integral expression for the total particle number of a free-field system:

$$N = \sqrt{\frac{8}{\pi}} \int_0^\infty \frac{\mathrm{d}t}{\sqrt{t}} V(t), \tag{5}$$

where $V(t)$ is the potential for the relaxed field. Now (4) can be solved numerically for interacting fields. Subsequently, $V(t)$ can be determined, and N can be computed by numerical integration. This technique presently only permits determination of the total particle number.

We now present our results using both the techniques discussed. Our results for the number distribution, computed in Coulomb gauge, are as follows. We have verified that for large $k_\perp$ our numerically obtained multiplicity agrees with the lattice analogue of the perturbative expression (3). At smaller $k_\perp$, the distribution softens and converges to a constant value, unlike its perturbative counterpart. Notably, this qualitative change of the distribution occurs at $k_\perp \sim \Lambda_s$. We tried to quantify this non-perturbative behavior by fitting the distribution to a variety of physically motivated functional forms. Surprisingly, we find that the shape of the distribution is closely reproduced by the Bose-Einstein form $n(k) = A/(\exp(\beta\omega_k) - 1)$, with the inverse temperature β of the order of 1 in units of Λ_s, and with ω_k corresponding to a free massive dispersion relation, with the mass of the order of $0.1\Lambda_s$. This is an unexpected result for a purely classical theory, whose meaning, beyond providing us with a convenient parametrization, is not yet clear.

Table 2 summarizes our findings for the total gluon number, written as $N = (\Lambda_s L/g)^2 f_N(\Lambda_s L)$. Note the good agreement between the two methods used to determine N (a more detailed analysis shows that the small difference between the two is likely an artifact of the Coulomb gauge). Note that, similarly to $f_E(\Lambda_s L)$, $f_N(\Lambda_s L)$ is a slowly-varying function. Based on Table 2, and making a naive extrapolation from the SU(2) to the SU(3) gauge group, we estimate the initial gluon number per unit rapidity as ~ 950 for central Au-Au collisions at RHIC energies, and as ~ 4300 for central Au-Au collisions at LHC. Let us again emphasize that, even though our figure for RHIC is not far from the recent experimental result, both the distribution and the total multiplicity are likely to be changed by further evolution of the field configuration.

$\Lambda_s L$	35.36	70.71	106.1	148.5	212.1	297.0
f_N (cooling)	.116(1)	.119(1)	.127(1)	.138(1)	.146(1)	.151(1)
f_N (Coulomb)	.127(2)	.125(2)	.135(1)	.142(1)	.145(1)	.153(1)

Table 2: Values of f_N vs $\Lambda_s L$ for the two definitions of the total particle number.

At the current stage of our project we are only able to make qualitative predictions. One obvious way to improve the accuracy is by replacing the SU(2) color group by the physical SU(3) one. Another is by allowing deviations from the strict boost invariance. These issues will be addressed in the future.

Acknowledgments

We would like to thank Larry McLerran and Al Mueller for very useful discussions. R. V.'s research was supported by DOE Contract No. DE-AC02-98CH10886. The authors acknowledge support from the Portuguese FCT, under grants CERN/P/FIS/1203/98 and CERN/P/FIS/15196/1999.

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